

# Adaptive Evolution: An Efficient Heuristic for Global Optimization

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## ABSTRACT

This paper presents a novel evolutionary approach to solve numerical optimization problems, called Adaptive Evolution (AEv). AEv is a new micro-population-like technique because it uses small populations (less than 10 individuals). The two main mechanisms of AEv are elitism and adaptive behavior. It has an adaptive parameter to adjust the balance between global exploration, local exploitation and elitism. Its two crossover operators allow a newly-generated offspring to be parent of other offspring in the same generation. AEv requires the fine-tuning of two parameters (several state-of-the-art approaches use at least three). AEv is tested on a set of 10 benchmark functions with 30 decision variables and it is compared with respect to some state-of-the-art algorithms to show its competitive performance.

## Categories and Subject Descriptors

G.1.6 [NUMERICAL ANALYSIS]: Optimization—*Unconstrained optimization*

## General Terms

Algorithms, Experimentation, and Performance

## Keywords

Numerical Optimization, Meta-Heuristics, Evolutionary Algorithms

## 1. ALGORITHM DESCRIPTION

AEv is a population-based stochastic optimizer based on a micro-population evolutionary algorithm. The two main mechanisms of AEv are: (1) elitism and (2) adaptive behavior. These mechanisms are mixed in a novel way within mutation, crossover and replacement operators. Algorithm 1 describes an AEv iteration. The features of AEv are:

1. Elitism, which has a great influence in the process. It is integrated in the crossover and replacement operators.
2. Adaptive parameters, which adjust: (1) elitism influences on the operators, (2) the step size used by the mutation operator, (3) the number of individuals generated in each crossover and, finally, (4) the restart mechanism. The three adaptive parameters are: (1) ambient pressure ( $\mathbf{C}$ ,  $C \in [1, P]$ ), (2)

step size for mutation operator ( $\vec{b}$ ,  $\vec{b}_i \in [0.0, 1.0]$ ,  $i \in [1, N]$ ) and (3) crossover balance ( $\mathbf{CR}$ ,  $CR \in [1, P - 1]$ ).

3. The mutation operator works in a similar way to those mechanisms used in a hill-climber search algorithms.
4. The two crossover operators allow an offspring to be a parent in the same generation. Parent selection is controlled by  $C$ , forcing the selection of the elite when  $C = P$ .
5. The replacement mechanism is combined with a reinitialization mechanism. Partial or total population restarts are allowed on each generations and controlled by  $C$ . Elitism is always ensured in replacement.
6. AEv requires the configuration of two user-defined parameters: Population size ( $\mathbf{P}$ ,  $P \geq 3$ ) and initial step size ( $\mathbf{B}$ ,  $B \in [0.0, 1.0]$ ) required by the mutation operator. In the experiments performed in this paper, AEv did not present a significant sensitivity to its parameter values.

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### Algorithm 1: Algorithm for any $g$ iteration of AEv.

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- 1 Recalculation of the three adaptive parameters;
  - 2 Copy  $X^g$  (population) into  $n^g$  (offspring);
  - 3 Perform mutation [5] to each  $n^g$  individual;
  - 4 **for**  $i = CR$  **To**  $P$  **do**
  - 5    $n_i^g = X_{[1, P-C+1]}^g + n_{[1, P]}^g + X_{[1, P]}^g$ ;
  - 6 **for**  $i = 1$  **To**  $CR$  **do**
  - 7    $c_1 = [0.0, 1.0]$ ,  $c_2 = [0.0, 1.0 - c_1]$ ,  $c_3 = 1.0 - c_2 - c_1$ ;
  - 8    $n_i^g = c_1 \times n_{[1, P-C+1]}^g + c_2 \times X_{[1, P-C+1]}^g + c_3 \times n_{[1, P]}^g$ ;
  - 9 Evaluate the new  $n^g$  individuals;
  - 10 Replace the first  $C$  individuals of  $X^{g+1}$  with the best ones of  $X^g \cup n^g$ . Replace the remaining individuals of  $X^{g+1}$  with random ones from  $n^g$ ;
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$C$  parameter controls the number of individuals to be affected by local exploitation vs. global exploration.  $\vec{b}$  is a vector of size  $N$  that contains the step size used by the mutation operator.  $CR$  controls the number of times each crossover operator is used.  $C$  and  $\vec{b}$  depend on the success of AEv search process.  $CR$  depends on the success of each crossover operator. The adaptive nature of AEv allows the algorithm to reach the vicinity of the global optimum with a high convergence speed even in complex problems.

The performance and features of AEv are shown by testing it in a set of 10 well-known functions with 30 variables taken from the specialized literature [2, 4] and detailed in Table 2. Table 1

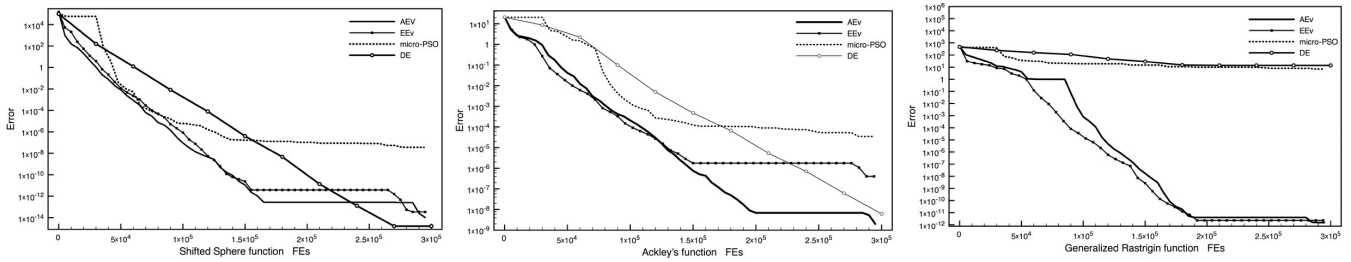


Figure 1: Convergence graphs of  $AEv_{P=5}$ , EEv,  $\mu$ -PSO and DE/rand/1/bin for  $f_{sph}$ ,  $f_{ras}$  and  $f_{ack}$  with  $N = 30$

Table 1: Results obtained by each compared algorithm. Normalized error values are shown in problems with 30 variables. Best results are remarked with boldface.

|            | $AEv_{P=5}$                  | EEv        | $AEv_{P=30}$                 | $\mu$ -PSO | $\mu$ -GA                    | DE                           | G3+PCX                       |
|------------|------------------------------|------------|------------------------------|------------|------------------------------|------------------------------|------------------------------|
| $F_1$      | $2.10E+00$                   | $5.48E+01$ | $3.13E+03$                   | $1.53E+06$ | $1.48E+15$                   | <b><math>1.00E+00</math></b> | $9.10E+00$                   |
| $F_2$      | $1.35E+09$                   | $7.66E+09$ | $8.16E+12$                   | $1.80E+10$ | $6.57E+14$                   | $2.05E+10$                   | <b><math>1.00E+00</math></b> |
| $F_3$      | $9.98E+02$                   | $8.66E+02$ | $2.52E+03$                   | $8.50E+03$ | $4.77E+04$                   | $3.39E+03$                   | <b><math>1.00E+00</math></b> |
| $F_4$      | $7.65E+02$                   | $1.07E+03$ | $5.74E+02$                   | $3.90E+01$ | $1.14E+02$                   | <b><math>1.00E+00</math></b> | $1.69E+03$                   |
| $f_{sch}$  | $4.28E+00$                   | $1.37E+01$ | $1.21E+01$                   | $1.28E+01$ | <b><math>1.00E+00</math></b> | $1.13E+01$                   | $3.42E+01$                   |
| $f_{ras}$  | <b><math>1.00E+00</math></b> | $1.23E+01$ | $1.49E+13$                   | $4.61E+13$ | $6.96E+13$                   | $4.71E+14$                   | $1.26E+15$                   |
| $f_{ros}$  | $8.96E+00$                   | $4.94E+00$ | <b><math>1.00E+00</math></b> | $1.26E+01$ | $3.88E+02$                   | $1.29E+00$                   | $1.41E+00$                   |
| $f_{ack}$  | <b><math>1.00E+00</math></b> | $2.48E+01$ | $1.50E+08$                   | $1.05E+04$ | $7.76E+08$                   | $2.05E+06$                   | $3.99E+09$                   |
| $f_{pen1}$ | <b><math>1.00E+00</math></b> | $2.87E+00$ | $1.01E+22$                   | $1.77E+14$ | $1.14E+24$                   | $1.71E+24$                   | $1.27E+25$                   |
| $f_{whit}$ | <b><math>1.00E+00</math></b> | $1.22E+00$ | $3.48E+00$                   | $6.27E+00$ | $3.65E+07$                   | $1.90E+01$                   | $3.92E+01$                   |
| $\mu()$    | <b><math>1.35E+08</math></b> | $7.66E+08$ | $1.01E+21$                   | $2.23E+13$ | $1.14E+23$                   | $1.71E+23$                   | $1.27E+24$                   |

presents a comparison of results among AEv with  $P = 5$  and  $P = 30$ ,  $\mu$ -GA [6],  $\mu$ -PSO [7], DE/Rand/1/Bin [3], G3+PCX [1] and a variant of AEv using only one crossover operator (EEv) to show AEv's efficiency. Fixed parameter values for all techniques on all the tests were used. 30 independent runs per each algorithm per each function were performed. All the experiments were run using a Pentium 4 PC with 512 MB of RAM, in C Linux environment. Figure 1 shows convergence graphs of  $AEv_{P=5}$ , EEv,  $\mu$ -PSO and DE/rand/1/bin for three test functions:  $f_{sph}$ ,  $f_{ras}$  and  $f_{ack}$  with 30 variables.

Test results obtained show that AEv provided a better performance with a small population  $P = 5$  in most of the test problems. It is a competitive approach, it has a smaller mean error value than the other techniques in the comparison. The two crossover approaches have a slightly better performance than the one crossover approach, more studies and refinement of mechanisms will be conducted on both approaches.

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Table 2: Test functions

### Unimodal functions

#### Separable

$F_1$  Shifted sphere function

#### Non-separable

$F_2$  Shifted Schwefel's problem 1.2

$F_3$  Shifted rotated high conditioned elliptic function

$F_4$  Shifted Schwefel's problem 1.2 w/ noise in fitness

### Multimodal functions

#### Separable

$f_{sch}$  Generalized Schefel's problem 2.26

$f_{ras}$  Generalized Rastrigin's function

#### Non-separable

$f_{ros}$  Generalized Rosenbrock's function

$f_{ack}$  Ackley's function

$f_{whi}$  Whitley's function

$f_{pen1}$  Generalized penalized function

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